

Please read these few pages before registering for the "the Principals of Moving Air." If you can successfully work through these 7 examples, your math is more than adequate to enable you able to focus on the fun parts of this course.

Item #3, says that an equation must remain equal on both sides, Item #7 reminds you to not mix the units of measure such as inches and feet etc, #8 reminds you of the general order of doing math calculations. #9 reminds you of how to use a fancy calculator, if you have one. (this is not required) All examples except # 3 are specific to moving air and illustrate the math steps with practical examples. Please don't be intimidated; be patient with yourself and walk through step by step.

1. Ducts, Perimeters, areas and volumes of a solid object

Perimeter (P) is the amount of sheet metal required to make the duct (not including the seam)

Round Duct (pipe)

$$P = 2\pi r \text{ and for } 7''\varnothing \text{ duct}$$

$$\begin{aligned} P &= 2 \times 3.14 \times \frac{7}{2} \\ &= 2 \times 3.14 \times 3.5 \\ &= 22.0 \text{ inches} \end{aligned}$$

Converting to feet

$$\begin{aligned} P &= 22 \text{ inches} \times \frac{1 \text{ foot}}{12 \text{ inches}} \\ &= \frac{22}{12} \text{ feet} \\ &= 1.8 \text{ feet (ft.)} \end{aligned}$$

Rectangular (or square) Duct

$$\begin{aligned} P &= (W + H) \times 2 \text{ and for } 14 \text{ inch} \times 8 \text{ inch duct} \\ &= (14 + 8) \times 2 \\ &= 22 \times 2 \\ &= 44 \text{ inches} \end{aligned}$$

Converting to feet

$$\begin{aligned} P &= 44 \text{ inches} \times \frac{1 \text{ foot}}{12 \text{ inches}} \\ &= \frac{44}{12} \text{ feet} \\ &= 3.7 \text{ feet (ft.)} \end{aligned}$$

The cross section area of a duct is the area enclosed by the duct which the air flows through.

Round Duct (pipe)

$$\begin{aligned} A &= \pi r^2 \text{ and for } 7''\varnothing \text{ duct} \quad \text{radius} = 3.5 \text{ inches} \\ &= 3.14 \times 3.5^2 \\ &= 3.14 \times 12.25 \\ &= 38.5 \text{ sq. in. or (in}^2\text{)} \end{aligned}$$

Converting to square feet (ft.²) 144 sq. in. = 1 sq. ft. same as 144 in.² = 1 ft.²

$$A = 38.5 \text{ in.}^2 \times \frac{1}{144}$$

$$= .267 \text{ square feet (or ft.}^2\text{)}$$

Rectangular (or square) Duct

$$A = W \times H \quad \text{and for 14 inch x 8 inch duct}$$

$$= 14 \times 8$$

$$= 112 \text{ sq. in.}$$

Converting to square feet (ft.²)

$$A = \frac{112 \text{ in.}^2}{144}$$

$$= .78 \text{ square feet (or ft.}^2\text{)}$$

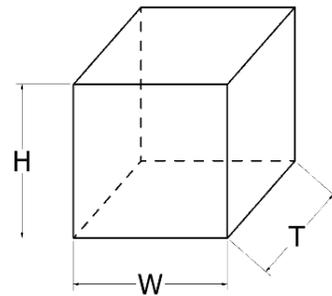
The volume of a duct is meaningless, but the volume of air moving per minute through a cross section of a duct is very important.

Volume of rectangular block

$$V = W \times T \times H \quad \text{and for } W = 14 \text{ in.}, T = 8'' \text{ and } H = 13''$$

$$= 14 \times 8 \times 13$$

$$= 1456 \text{ cubic inches}$$



Converting to cubic feet (ft.³)

$$V = \frac{1456 \text{ in.}^3}{12 \times 12 \times 12}$$

$$= .84 \text{ cubic feet (or ft.}^3\text{)}$$

Volume of cylindrical block

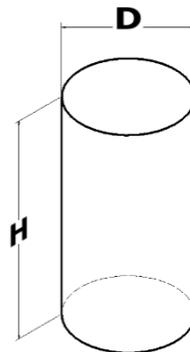
$$V = \pi r^2 \times H \quad \text{and for Diameter} = 7 \text{ in. \& } H = 13'' \text{ radius} = 3 \frac{1}{2}''$$

$$= \pi \times (3.5 \text{ in})^2 \times 13 \text{ cubic inches (or in.}^3\text{)}$$

$$= 3.14 \times 3.5^2 \times 13 \text{ in.}^3$$

$$= 3.14 \times 12.25 \times 13$$

$$= 500 \text{ cubic inches or (in}^3\text{)}$$



Converting to cubic feet (ft.³)

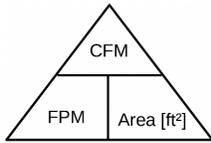
$$V = \frac{500 \text{ in.}^3}{12 \times 12 \times 12}$$

$$= \frac{500}{1728}$$

$$= .29 \text{ cubic feet (or ft.}^3\text{)}$$

2) Multiply & Divide Triangle

POMA uses this simple triangle method as a math reminder on how handle 3 number equations. For example:



This triangle contains 3 abbreviated terms:

cfm = volume flow rate in cubic feet per minute

fpm = velocity in feet per minute

area = duct cross section area

Using the Triangle:

cfm = fpm x area or

fpm = $\frac{\text{cfm}}{\text{area}}$ or

area = $\frac{\text{cfm}}{\text{fpm}}$

3. Balance Math rule of thumb. Like bookkeeping, any change made to one side of an equation, must be made equally to the other.

4) Ratios

Fan Law #1, says that cfm and rpm are directly related. This means that doubling the cfm will require doubling the fan wheel rpm.

$\frac{\text{cfm}_2}{\text{cfm}_1} = \frac{\text{rpm}_2}{\text{rpm}_1}$ cfm_2 is the flow after you change the fan rpm
 cfm_1 is the flow rate initially, before the change

By multiplying both sides of this formula by cfm_1 , it can be re-written as follows.

$$\frac{\text{cfm}_2}{\text{cfm}_1} \times \text{cfm}_1 = \frac{\text{rpm}_2}{\text{rpm}_1} \times \text{cfm}_1$$

$$\text{cfm}_2 = \frac{\text{rpm}_2}{\text{rpm}_1} \times \text{cfm}_1$$

Example: if the initial fan rpm was 800 and air flow rate was 1400 cfm, if you raise the rpm to 1000, what will be the new cfm?

$$\text{cfm}_2 = \frac{1000}{800} \times 1400$$

$$= 1.25 \times 1400$$

$$= 1725$$

5a. Squares and square roots (squares)

Fan Law #2 says that the static pressure drop through duct and across filters and through coils follows a square law. This means to double the air flow through a duct system will require quadrupling (4 times) the static pressure.

$\frac{\text{Ps}_2}{\text{Ps}_1} = \frac{\text{cfm}_2}{\text{cfm}_1}$ **squared** Ps_2 is the static pressure after increasing the flow rate
 Ps_1 is the static pressure initially before making the change

By multiplying P_{s1} to both sides of this formula, it can be re-written as follows. (Note: both sides remain equal)

Where words squared, square root, cubed and cube root are shown in red, here in these examples, this means divide the preceding numerator by denominator first.

$$\frac{P_{s2}}{P_{s1}} \times P_{s1} = \frac{cfm_2 \text{ squared}}{cfm_1} \times P_{s1} \quad \text{or} \quad = \frac{(cfm_2)^2}{(cfm_1)^2} \times P_{s1}$$

$$P_{s2} = \frac{cfm_2 \text{ squared}}{cfm_1} \times P_{s1} = \frac{(cfm_2)^2}{(cfm_1)^2} \times P_{s1}$$

Example: if the initial system flow rate was 1400 and initial system pressure (static pressure = P_s) was .30" WC, if you need to raise the flow rate to 1725 cfm, the new static pressure will rise to: note 'or': there are 2 ways to work this out.

$$\begin{aligned} P_{s2} &= \frac{1725 \text{ squared}}{1400} \times P_{s1} & \text{'or'} & = \frac{(1725)^2}{(1400)^2} \times P_{s1} \\ &= \frac{1725 \text{ squared}}{1400} \times .30 & & = \frac{(1725)^2}{(1400)^2} \times .30 \\ &= (1.232)^2 \times .30 & & = \frac{2,975,625}{1,960,000} \times .30 \\ &= 1.518 \times .30 & & = 1.518 \times .30 \\ &= .455 & & = .455 \end{aligned}$$

5b. Squares and square roots (square roots)

Using the square formula backwards requires understanding square roots. For example, if you increased the pressure (static pressure) from .30" to .40" what would be the new flow rate based on the initial 1400 cfm?

$$\frac{P_{s2}}{P_{s1}} = \frac{cfm_2 \text{ squared}}{cfm_1} \quad P_{s2} \text{ is the static pressure after increasing the flow rate}$$

$$P_{s1} \text{ is the static pressure initially before making the change}$$

$$\frac{cfm_2 \text{ squared}}{cfm_1} = \frac{P_{s2}}{P_{s1}} \quad (\text{Reversing the left with the right side of the equation})$$

Square root both sides of equation. The square root of a square cancels out

$$\frac{cfm_2}{cfm_1} = \frac{P_{s2} \text{ square root}}{P_{s1}} \quad \text{or} \quad = \frac{(P_{s2}) \text{ sq. root}}{(P_{s1}) \text{ sq. root}}$$

$$\frac{cfm_2 \times cfm_1}{cfm_1} = \frac{P_{s2} \text{ square root}}{P_{s1}} \times cfm_1 = \frac{(P_{s2}) \text{ sq. root}}{(P_{s1}) \text{ sq. root}} \times cfm_1$$

$$\frac{cfm_2}{.30} = \frac{.40 \text{ square root}}{.30} \times 1400 = \frac{.40 \text{ sq. root}}{.30 \text{ sq. root}} \times 1400$$

$$= 1.3333 \text{ square root} = \frac{.632}{.548} \times 1400$$

$$= 1.155 \times 1400 = 1.155 \times 1400$$

$$= 1617$$

$$= 1617$$

6a. Cubes and cube roots (cubes)

Fan law #3 is the only time cubes and cube roots are required. This means to double the air flow through a duct system requires 8 times the power. Wow!

$\frac{Hp_2}{Hp_1} = \frac{cfm_2}{cfm_1}$ cubed Hp_2 is the horse power required after increasing the flow rate
 Hp_1 is the horse power required initially before making the change

By multiplying Hp_1 to both sides of this formula, it can be re-written as follows; while both sides remain equal.

$$\frac{Hp_2}{Hp_1} \times Hp_1 = \frac{cfm_2}{cfm_1} \text{ cubed} \times Hp_1 \quad \text{or} \quad = \frac{(cfm_2)^3}{(cfm_1)^3} \times Hp_1$$

$$Hp_2 = \frac{cfm_2}{cfm_1} \text{ cubed} \times Hp_1 \quad = \frac{(cfm_2)^3}{(cfm_1)^3} \times Hp_1$$

Example: The initial system flow rate was 1400 which required 1/4 horse power, if you need to raise the flow rate to 1725 cfm, the new motor would need to be:

$$\begin{aligned} Hp_2 &= \frac{1725}{1400} \text{ cubed} \times 1/4 & \text{or} & = \frac{(1725)^3}{(1400)^3} \times Hp_1 \\ &= 1.232 \text{ cubed} \times 1/4 & & = \frac{5.132 \text{ billion}}{2.74 \text{ billion}} \times 1/4 \\ &= 1.87 \times 1/4 & & = 1.87 \times 1/4 \\ &= .47 & & = .47 \end{aligned}$$

6b. Cubes and cube roots (cube roots)

Using this formula backwards requires understanding cube roots. For example if you increased the horse power from 1/4 to 1/3, how much could you increase the flow rate based on having an initial 1400 cfm?

$\frac{Hp_2}{Hp_1} = \frac{cfm_2}{cfm_1}$ cubed Hp_2 is the static pressure after increasing the flow rate
 Hp_1 is the static pressure initially before making the change

$\frac{cfm_2}{cfm_1} \text{ cubed} = \frac{Hp_2}{Hp_1}$ Cube root both sides of equation. The cube root of a cube cancels out

$$\frac{cfm_2}{cfm_1} = \frac{Hp_2}{Hp_1} \text{ cube root} \quad \text{or} \quad = \frac{(Hp_2) \text{ cube root}}{(Hp_1) \text{ cube root}}$$

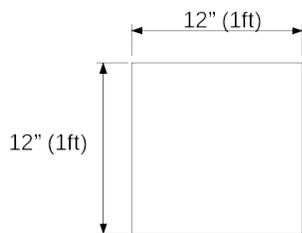
By multiplying cfm_1 to both sides of this formula, it can be re-written as follows. (while both sides remain equal)

$$\frac{cfm_2}{cfm_1} \times cfm_1 = \frac{Hp_2}{Hp_1} \text{ cube root} \times cfm_1 \quad = \frac{(Hp_2) \text{ cube root}}{(Hp_1) \text{ cube root}} \times cfm_1$$

$$\begin{aligned}
 \underline{\text{cfm}_2} &= \frac{1/3 \text{ cube root}}{1/4} \times 1400 &= \frac{1/3 \text{ cube root}}{1/4 \text{ cube root}} \times 1400 \\
 &= 1.3333 \text{ cube root} \times 1400 &= \frac{.6933}{.6299} \times 1400 \\
 &= 1.10 \times 1400 &= 1.10 \times 1400 \\
 &= 1540 \text{ (Note: not much increase)} = 1540
 \end{aligned}$$

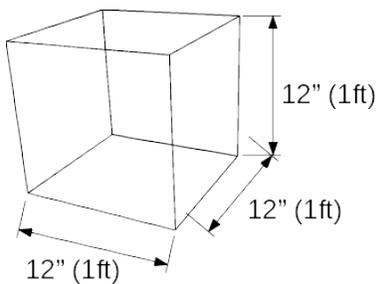
7. Units

The conversion of units is critical, a large number of mistakes in the real world are caused by conversion errors. (crash landing a Boeing 767 @ Gimley, Manitoba) Keep the units in the calculation, at least initially. This will help convert all the measurements into the correct units and prevent multiplying feet by inches!



$$\begin{aligned}
 \text{Area in in}^2 &= 12" \times 12" = 144 \text{ in}^2 \\
 \text{Area in ft}^2 &= 1' \times 1' = 1 \text{ ft}^2 \\
 1 \text{ ft}^2 &= 144 \text{ in}^2
 \end{aligned}$$

$$\begin{aligned}
 \text{Volume in in}^3 &= 12" \times 12" \times 12" = 1728 \text{ in}^3 \\
 \text{Volume in ft}^3 &= 1' \times 1' \times 1' = 1 \text{ ft}^3 \\
 1 \text{ ft}^3 &= 1728 \text{ in}^3
 \end{aligned}$$



Example) Calculate the airflow (cfm) given the velocity is 500 fpm in a 24" x 8" duct.

Given: $v = 500 \text{ fpm}$ $A = 24" \times 8"$

Find: Airflow in cfm

Airflow = Area x velocity

$\text{cfm} = (24" \times 8") \times 500 \text{ fpm}$ You should not multiply inches by feet, first convert the duct area to square feet.

Duct area in square feet = 24 inches x 8 inches = 192 square inches

$$= 192 \text{ square-inches} \times \frac{1 \text{ square foot}}{144 \text{ square-inches}}$$

$$= \frac{192}{144} \text{ square feet} = 1.333 \text{ square feet} \quad \text{or sq. ft.} \quad (\text{or ft}^2)$$

Now calculate air flow (cfm) if velocity = 500 cubic feet per minute

CFM = Area x Velocity (no need to remember formulas, use the math 'triangle')

$$= 1.333 \text{ square feet} \times \frac{500 \text{ feet}}{1 \text{ minute}}$$

$$= 1.333 \times 500 \frac{\text{cubic feet}}{1 \text{ minute}}$$

$$= 667 \text{ cubic feet per minute (cfm)}$$

Note: For conversion problems, check the Common Factors Conversion Table.

8. Order of Operations (BEDMAS)

It is important to do math problems in the correct order. To help remember that, use the acronym "BEDMAS" which stands for:

1. Brackets (x + y)
2. Exponents x^n {and roots \sqrt{x} }
3. Division and Multiplication
4. Addition and Subtraction

9. Calculators, only read this if you use a scientific type, not required in this course

Note: If you enter information into a calculator, make sure the brackets are in the correct place and place the exponent outside the brackets.

* if your calculator does not have a square root (somewhat common) guess the root, square it then check if the guess was correct. 3 or 4 tries will give you a very close answer.

Example for calculator)

The symbol $\sqrt{\quad}$ implies $\sqrt[2]{\quad}$ known as a square root, but it is possible to have a number 3 in that top position.

* if your calculator does not have a cube root (not common) guess the root, cube it then check if the guess was correct. 3 or 4 tries will give you a very close answer.

Most calculators have an X^2 button and y^x (which may appear as $^{\wedge}$).

ex) $2^3 = 2 \times 2 \times 2 = 8$

Calculator:	2	y^x	3	enter
-------------	---	-------	---	-------

Some calculators will have $\sqrt[3]{\quad}$ as a button, but it may appear as $\sqrt[x]{\quad}$ and you must input the number you want for X before pressing the button.

3	$\sqrt[x]{\quad}$	64	enter
---	-------------------	----	-------